

Validated dynamic energy model for a Stirling engine μ -CHP unit using field trial data from a domestic dwelling

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ABSTRACT

This paper presents the development and validation of a dynamic energy performance model for a WhisperGen Stirling engine micro-CHP (μ -CHP) Mk 4 unit. The model is based on empirical equations for different system characteristics (start-up, continuous operation and shutdown) developed using measured performance data. The model was implemented using MatLab program and its validation was undertaken against measured performance data from a unit installed in a dwelling under field trial in Northern Ireland. Given a heat demand profile for a domestic dwelling, the model can be used to predict the heat and electricity output. Using heat and electricity data for a dwelling over a period of 1 month, the model over-predicted the total electrical and thermal energy output from the μ -CHP unit by 1.4% and 4.2% respectively. The model predicted the electrical power output with a mean absolute error of 7.4% and a cumulative over-prediction of 4.7%. The thermal power output was predicted with a mean absolute error of 7.2% and a cumulative under-prediction of 3.7%.

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1. Introduction

Combined heat and power (CHP or cogeneration) units simultaneously produce heat and electrical power through the combustion or catalysis (for fuel cells) of fossil fuels such as natural gas. The systems are highly efficient, often converting more than 80% of input fuel energy into electricity and heat. However, their high efficiency is dependent on a simultaneous demand for heat and power at local level. CHP is most commonly deployed in industry where there is a complimentary electrical and process heat demand, and in district heating systems where electricity is fed onto the grid and heat is distributed to domestic customers through insulated pipe work. The use of CHP is recognised as an important 'near term low carbon technology' by the International Energy Agency for reducing greenhouse gas GHG emissions [1].

Micro-CHP (μ -CHP) systems typically have electricity outputs between 1 and 10 kW and heat outputs in the range of 1–35 kW. They have been developed specifically for individual domestic and small commercial buildings which have a natural gas supply where they can be easily retrofitted without the need for expensive district heating pipe work. Power is either used in the dwelling or exported to the grid and the heat generated is used for space heating and hot water. Where a cooling load exists, it is technically feasible to use

the heat output to drive absorption chillers [2]. This flexibility of distributed μ -CHP systems to provide local heat, hot water, cooling and electricity is regarded as a significant benefit of the technology [3].

The growing worldwide demand for less polluting forms of energy has driven the development of the μ -CHP technologies in the residential sector. The most common μ -CHP technologies are fuel-cells, external combustion (Stirling) engines and internal combustion engines. The latter is presently in the market place [4].

The residential sector currently accounts for approximately 25% of direct primary energy used in developed economies such as the EU. It is claimed that large scale μ -CHP adoption would significantly reduce this energy use [5] due to the displacement of inefficient electricity production using centralised plants and the avoided transmission and distribution losses associated with embedded generation. Policies aimed at reducing CO₂ emissions are central to the development and deployment of future power systems. The EU's strategic energy technology plan (SET-Plan), for example envisages a 20% reduction in CO₂ emissions by 2020. Near term low carbon technologies with mass-market potential such as μ -CHP [6] are therefore attractive to policymakers. However, evidence of their benefits should be obtained before developing policies for their promotion.

A number of researchers have developed models that can be used to predict the thermal and electrical energy outputs as well as the economic performances of different types of μ -CHP systems. The models are however system specific and do not apply

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Nomenclature

HPR	heat to power ratio (dimensionless)
Q_{th}	useful thermal power output (kW)
P_e	electrical power output (kW)
Q_f	primary fuel consumed (kW)
η_0	overall efficiency (%)
η_{th}	thermal efficiency (%)
η_e	electrical efficiency (%)

to other μ -CHP units that have different operating characteristics. The models are summarised below.

Ferguson et al. [7] developed an empirical model for Stirling engines (SE) and internal combustion engine (ICE) μ -CHP systems based on laboratory data. Onovwiona et al. [8] developed a parametric model for ICE μ -CHPs with thermal and electrical storage for residential applications which can be used for techno-economic assessment as well as system optimisation.

Cho et al. [9] developed an energy dispatch algorithm that provides the operational signal to the μ -CHP components to operate the system in the most economical manner by minimizing the cost of energy efficiency constraints for each component.

Bernd [10] investigated the performance of four commercially available μ -CHP units using laboratory tests as well as both steady and unsteady state benchmark tests in regard to the German environmental label.

Barelli et al. [11] developed a complete dynamic model of a residential μ -CHP system based on proton exchange membrane fuel cell (PEMFC) technology including fuel processor, heat exchangers, humidifier and auxiliary hot water boiler. The model was used to investigate the behaviour of the system when meeting typical variable residential electrical and thermal loads.

De Paepe et al. [12] analysed the operational parameters of two gas engines, two Stirling engines and a fuel cell μ -CHP units. Dynamic simulation was used to compare the performance of the μ -CHP systems to a dwelling using a conventional natural gas boiler with electricity imported from the grid. Their results showed that if all the μ -CHP systems are well sized, they would result in a reduction of primary energy use and that the gas engines had the best performance. Under favourable economic conditions, they reported that only the gas engines had positive internal rate of return on investment of less than 5%.

Peacock and Newborough [13] investigated the effects of applying combined heat and power systems based on Stirling engines and fuel cells to single UK dwellings using heat and power demand data recorded at 1-min intervals for a full year. They reported that the control regime has a major influence on both economic and CO₂ savings for Stirling engine systems due to their high heat-power ratio.

Ren and Gao [2] analysed the performance of two typical μ -CHP units with a gas engine and fuel cell for residential buildings. Two different operating modes including minimum-cost operation and minimum-emission operation were taken into consideration by employing a plan and evaluation model for residential μ -CHP systems.

TeymouriHamzehkolaei and Sattari [14] carried out a technical and economic feasibility study of using μ -CHP in different climatic zones in Iran. Ben Maalla and Kunsch [3] analysed the potential for the diffusion of μ -CHP installations as a substitute for centralised electricity generation and local boilers in the residential sector using simulation analysis with System Dynamics.

Voorpoels and D'haeseleer [15] reported that the transient behaviour of an internal combustion engine (ICE) during cold start operation has significant effect on the heat production, whereas

for electricity generation, the transient behaviour is negligible. They modelled the cold start behaviour of an ICE using regression equations developed using experimental data on the system's performance. They considered different start-up intervals after shutdown notably: 1 h, 2 h, 4 h and cold start.

Kelly and Beausoleil-Morrison [16] developed a simulation model (Annex 42 model) that can be used to characterise the thermal and electrical performance of combustion-based micro-cogeneration devices such as solid oxide and polymer exchange membrane fuel cells (SOFC and PEM), and internal combustion and Stirling engine units (ICE and SE). Their model which was calibrated to represent the performance of a particular Stirling engine μ -CHP unit is suitable for use in whole-building simulation tools because of its parametric design and empirical nature. Despite being one of the few existing combustion engine models suitable for use in whole-building simulation tools, the Annex 42 model is complicated and has some limitations which include assumption of no heat generation during shut-down; relates the fuel flow and net electricity produced at start-up to the engine temperature; represents the steady state efficiency by a 27-term polynomial of the coolant flow rate, coolant inlet temperature and net electrical output.

Lombardi et al. [17] reported that studies carried out to assess the overall performance of SE μ -cogeneration devices in different climates using the Annex 42 model highlighted the importance of accurately modelling start-up and shut-down behaviour. They however, noted that the data used in the calibration of the Annex 42 model was not ideal since it was collected during typical operating cycles rather than steady-state conditions. They therefore used a fit for purpose data set to propose improvements in the treatment of steady-state, start-up, and shut-down sequences of operation. Finally, they proposed a simpler form of the steady-state efficiency which relies on empirical data for the coolant flow rate, coolant inlet temperature and net electrical output.

Given that μ -CHP systems are either at a market introduction or pre-commercial stage, limited work has been undertaken on modelling their installed energy and financial performances. One approach is to develop individual techno-economic models using field performance data [8]. Such models developed can be used to estimate technology performances under dwelling demand profiles representing different dwellings or the effect of energy efficiency measures. Any such models developed are empirical and specific to commercially available systems with Otto-cycle internal combustion engines. However, the only empirical model developed for a Stirling engine μ -CHP (with a net electrical output of 750 W and a thermal output of 5 kW) is the Annex 42 model [16] that was modified by Lombardi et al. [17].

The objective of this paper is therefore to develop and validate an energy performance model that can accurately predict the fuel consumed, heat and electricity generated by a WhisperGen Stirling engine μ -CHP Mk 4 unit (with a net electrical output of 1 kW and a net thermal output of 8 kW) linked to a heating and electrical system which is characteristic of domestic dwellings.

2. Methodology

The methodology employed in this paper to develop and validate the dynamic energy model for the Stirling engine μ -CHP unit consisted of using a month's performance data collected at 5-min intervals. The data was sourced from a field trial of a WhisperGen Mk 4 μ -CHP unit installed in a semi-detached house to provide energy for domestic hot water, space heating and electrical systems. The data were analysed to identify modes of operation based on visual inspection of plots of heat and electrical outputs. This led to identification of three main modes: start-up, continuous

operation and shutdown. It also led to identification of different start-up characteristics depending on time since last shut-down. The data were then used to compute the thermal, electrical and overall efficiencies as well as the heat to power ratio during start-up and continuous operation.

Empirical models were developed by fitting polynomial equations to heat and electrical power as a function of time for the three main modes. The polynomial equations were then combined using an algorithm developed for the entire operation of the μ -CHP unit and implemented in MatLab program. Since the μ -CHP unit is heat led, the heat demand profile for the dwelling was extracted from the heat output data and used to run the MatLab program. The data were used to verify that the predicted heat and electrical power output from the MatLab program corresponded to the measured data used to develop the polynomial equations.

Data used to validate the energy model for the μ -CHP unit were different from that used to develop the polynomial equations for the different operating modes. The heat demand profile used to run the model was obtained from the data used to validate the model. Heat and electrical power output predicted by the model were validated against measured data using percentage mean error and percentage mean absolute error while the total electrical and thermal energy output predicted by the model were validated using the cumulative error.

2.1. System description

The house evaluated in this study is a typical 4 bedrooms semi-detached single family residential dwelling with a floor area of 140 m². The dwelling was built in the 1980s, was well-insulated and previously equipped with a conventional boiler. It complied with the building regulations in force in Northern Ireland at that time. The house had four occupants and the principal heat demand of the building was for space heating and for a constant supply of hot water. Parameters measured during the field trial that are considered essential in developing a model for the μ -CHP system include fuel consumed, electrical and thermal power output.

The unit used in the field trial was a prototype WhisperGen Stirling engine μ -CHP Mk 4 unit (hereafter referred to as “the unit”) manufactured by WhisperTech, New Zealand (8 kW thermal and 1 kW electrical). This was the only commercially available unit at the time of this field trial which was suitable for installation in a domestic dwelling.

The unit is a four cylinder double acting external combustion engine. No fuel or air mixture is taken into the cylinders which are filled with a fixed volume of nitrogen gas. This results in a relatively clean and quiet operation when compared to commercialised internal combustion engine, thus making it more suitable for installation inside a domestic dwelling. Heat to the four nitrogen-filled engine cylinders is provided by a burner situated on top of the cylinders. This is heated by a flow of burning natural gas and air causing it to expand and it contracts when cooling water flows through the cylinder. Expansion and contraction of the nitrogen gas in the engine cylinders causes the pistons within the cylinders to move with the reciprocating motion converted to a rotary generator by a unique ‘wobble yoke’ mechanism which converts the reciprocating motion of the pistons into rotary motion [18]. Heat gained through the water can be used for domestic space and hot water heating.

2.1.1. Characterising modes of operation

Veitch and Mahkamov [19] characterised the performance of a WhisperGen Stirling engine μ -CHP operated under laboratory conditions using a time step of 1 min. Two sets of field trial data measured at 5 min and 30 min intervals sourced from a WhisperGen Mk 4 μ -CHP unit linked to the domestic hot water, space heating and electrical systems of a three-bedroomed, semi-detached

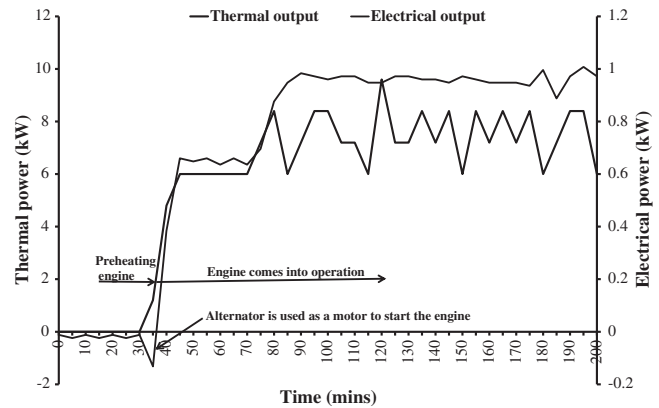


Fig. 1. Start-up characteristics of the μ -CHP unit.

house were available for this study. Performance data at 30 min intervals were found to be unsuitable for analysing system characteristics because they do not capture rapid changes in system parameters during start-up and shut down which are important to accurately model the system's performance. The data collected at 5 min intervals showed significant variations in the heat and electrical power output over time during the first 25–65 min of operation, with relatively steady state conditions thereafter.

Three operating conditions were identified (from a visual inspection of heat and electrical power output plots) from the 5-min field performance data that affect the operation of the μ -CHP unit. The operating conditions include start-up; shut down; and continuous operation. The combined start-up, continuous operation and shut down phases are referred to as modulated operation. Start-up describes the period when the unit comes on and starts generating both heat and electrical power until it attains a period of maximum power output, which is known as continuous operation. After a period of continuous operation, the unit shuts down when there is no heat demand.

2.1.2. Start-up

Fig. 1 shows electrical and heat outputs from the μ -CHP unit during start-up. The start-up can be divided up into two phases based on electrical power output. The first is the engine preheating phase where about 150 W of electricity powers the alternator which serves as a motor to start the engine; a small amount of electricity is consumed by the unit during this phase. For the first 10 min or so, the unit begins to produce heat and power which increase linearly and a 30 min ‘plateau’ ensures where heat and power output remain constant at approximately 6.0 kW and 0.7 kW respectively before again rising to their maximum outputs. The second phase is the operational phase when the thermal and electrical power outputs increase nonlinearly to a maximum of approximately 7.8 kW and 1.0 kW respectively.

Clucas and Raine [20] explain that during start up times above 1 h after shutdown, the piston seal cools and shrinks, creating a seal that gives good starting conditions. However, after running for several minutes it heats up, expands and the seal leaks, stalling the engine. This causes the electrical power output to plateau at around 0.6 kW for a period of time. However, the activation pressure as a result of the expanding nitrogen heated in the cylinder restores the seal and the engine then reaches maximum operational speed.

2.1.3. Continuous operation

The continuous operating condition occurs between unit start-up and shutdown. This is an operating condition when the μ -CHP unit continuously generates maximum heat and electricity over a given time interval of consistent domestic heat demand. Fig. 2

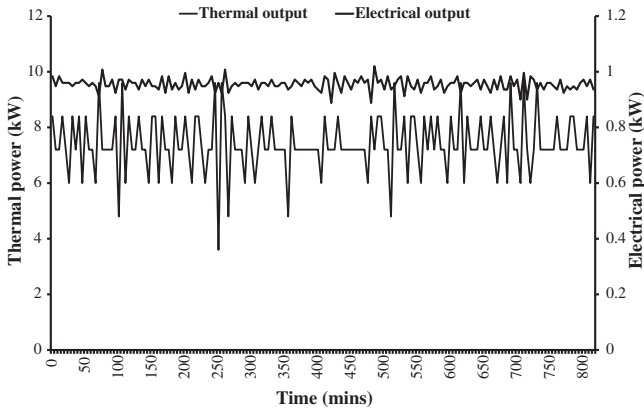


Fig. 2. Thermal and electrical power output during continuous operation.

shows the thermal and electrical power generated and operating cycle length during a period of continuous operation, where the electrical power output remains fairly constant at about 0.96 kW and the thermal power output fluctuates from 6–8.4 kW.

2.1.4. Shutdown

Fig. 3 shows the shutdown characteristics of the μ -CHP unit. It consists of a single phase during which the engine is stopped in a controlled manner allowing the high thermal mass in the hot cylinder to be gradually dissipated into the domestic heating system. The pump and fan continue running for some tens of minutes after fuel combustion has stopped in order to transfer useful heat into the heating system and prevent unit overheating. An electrical pump and fan load of 140 W can continue for up to 45 min after combustion has stopped.

2.2. System performance

In cogeneration applications for heat and power production, the heat to power ratio (HPR) is given by [8]

$$\text{HPR} = \frac{Q_{\text{th}}}{P_{\text{e}}} \quad (1)$$

The thermal and electrical efficiencies of the μ -CHP unit are respectively

$$\eta_{\text{th}} = \frac{Q_{\text{th}}}{Q_{\text{f}}} \times 100 \quad (2)$$

$$\eta_{\text{e}} = \frac{P_{\text{e}}}{Q_{\text{f}}} \times 100 \quad (3)$$

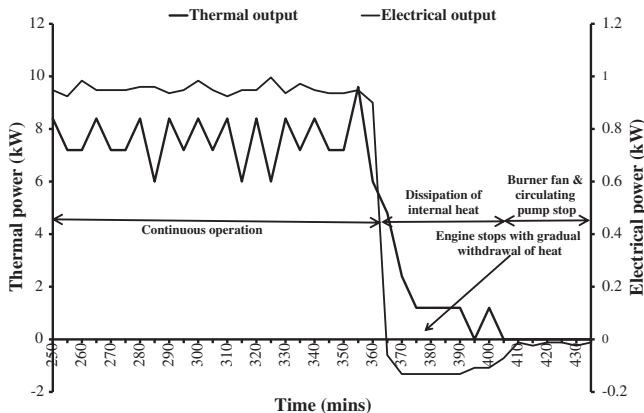


Fig. 3. Shutdown characteristics of the μ -CHP unit.

Thus, the overall efficiency (η_0) of the μ -CHP unit is given as [8]

$$\eta_0 = \frac{Q_{\text{th}} + P_{\text{e}}}{Q_{\text{f}}} \times 100 \quad (4)$$

The time-series data collected were divided into each of the three operational conditions identified above: start-up; shut down; and continuous operation. Eqs. (1)–(4) were used to calculate the heat to power ratio, thermal, electrical and overall efficiencies for each of the system's characteristics under each of the operating conditions. Table 1 presents a summary of the μ -CHP characteristics during start-up and continuous operation. Shut down characteristics are not calculated since no fuel is used during this period, and efficiencies are not relevant.

Analysis of the start-up periods (from start of pre-heating to constant heat and power output) shows that the thermal efficiency increased from 58.8% to 61.8% for start-up times less than 1 h and greater than 2 h after shutdown respectively and then reached a maximum of 67.7% during continuous operation. The lower efficiency is as a result of the μ -CHP unit being at a higher temperature due to the shorter time interval after shutdown (for $t_s < 1$ h) resulting both in lower temperature differences and heat transfer rates into the domestic heating system. A maximum efficiency of 67.7% was recorded during continuous operation.

The electrical efficiency decreased from 7.1% to 6.4% for start-up times less than 1 h and greater than 2 h after shutdown respectively and then reached a maximum of 8.7% during continuous operation. The HPR was 7.8 during continuous operation and it increased from 8.2 to a maximum of 9.6 for start-up times less than 1 h and greater than 2 h after shutdown respectively.

The quantity of fuel consumed can be calculated using Eq. (2), modelled thermal power output and thermal efficiency for different modes of operation detailed in Table 1. Similarly, the quantity of fuel consumed can be calculated using Eq. (3), modelled electrical power output and electrical efficiency for different modes of operation detailed in Table 1.

3. Empirical modelling

A month's performance data at 5-min intervals obtained from a field trial of a WhisperGen Stirling engine μ -CHP Mk 4 unit in the UK by the Carbon Trust were used in the analysis. The measured data consisted of gas consumed as well as electrical and thermal power generated by the μ -CHP unit. Data were analysed and the characteristics of the μ -CHP unit were calculated and presented.

Based on the findings by Voorpools and D'haeseleer [15] that energy output from a μ -CHP is dependent on the time since it was last operating, the start-up characteristics of the Stirling engine μ -CHP in this study were studied, 1 h, 2 h, 4 h after previous operation as well as from a 'cold start' (greater than 4 h). Thermal and electrical power data were divided into the corresponding time intervals: less than 1 h, 1–2 h, 2–4 h and more than 4 h. Average start-up profiles were generated and best-fitting techniques were used to identify a set of representative polynomial equations for each time interval for both thermal and electrical outputs. Similarly, equations for continuous operation and shut down were generated and these were then used to develop an energy input–output model for the system for all operational conditions with a 5-min time step.

Since μ -CHP systems are heat-led, the heat demand profile in Fig. 5 was then used to run the μ -CHP system model with thermal and electrical power output predicted at 5 min intervals. Results from the system model were compared with measured field performance data and validation was carried out using two statistical parameters; percentage mean absolute error (PMEA) and percentage mean error (PME) described in Eqs. (5) and (6) respectively. Using the validated system model and heat demand profile for a

Table 1
 μ -CHP unit characteristics during start-up and continuous operation.

μ -CHP characteristics	Interval between shutdown and start-up (t_s)				Continuous operation
	$t_s < 1$ h	$1 < t_s \leq 2$ h	$2 < t_s \leq 4$ h	$t_s > 4$ h	
Thermal efficiency (%)	58.8	60.5	61.8	61.8	67.7
Electrical efficiency (%)	7.1	6.5	6.4	6.4	8.7
Overall efficiency (%)	66.0	67.0	68.2	68.2	76.4
HPR	8.2	9.3	9.6	9.6	7.8
Approximate time to attain continuous operation for electricity generation (min)	35	45	50	55	
Approximate time to attain continuous operation for heat generation (min)	30	45	65	65	

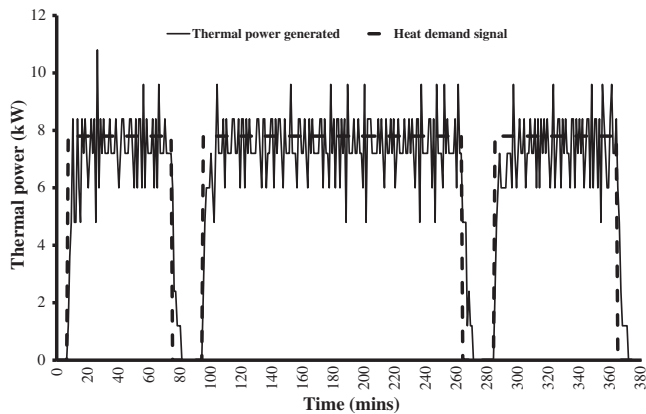


Fig. 4. Sample heat demand signal and thermal power generated by the μ -CHP unit.

house, the long-term performance of the μ -CHP system installed in the house can be predicted.

3.1. Heat demand signal

Fig. 4 shows a sample heat demand signal for a domestic dwelling and the thermal power generated by the μ -CHP unit. It is seen that heat is required for different intervals after shutdown over varying periods.

3.2. Electrical power output

Fig. 5 shows plots of electrical power output by the μ -CHP unit for different start-up times after shutdown. It is seen that for start-up less than 1 h after shutdown, the electrical power output increases fairly smoothly and then reaches maximum output about 30–35 min after. For start-up intervals above 1 h after shutdown, the electrical power output increases rapidly to approximately 0.6 kW and then remains fairly constant before ramping up to the maximum of 0.96 kW. The time taken to attain maximum electrical

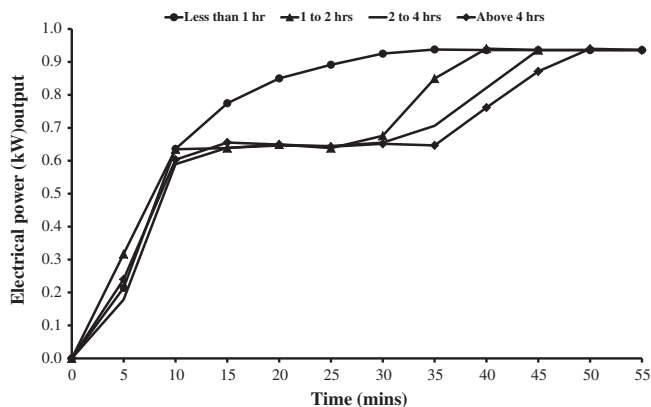


Fig. 5. Electrical power output for different start-up times after shutdown.

Table 2
 Duration, correlation equations and coefficients for electricity output modelling.

Duration	Equation	R^2
<i>Less than 1 h</i>		
0–10 min	$P_e = 0.0036t^2 + 0.0216t$	1.00
15–35 min	$P_e = -0.00036t^2 + 0.0276t + 0.456$	1.00
<i>1–2 h</i>		
0–10 min	$P_e = 0.0636t - 0.00024$	1.00
15–30 min	$P_e = 0.00024t^2 - 0.0096t + 0.7308$	0.74
35–45 min	$P_e = 0.0108t + 0.486$	0.86
<i>2–4 h</i>		
0–10 min	$P_e = 0.0048t^2 + 0.012t$	1.00
15–30 min	$P_e = -0.000036t^2 - 0.0006t + 0.642$	0.83
35–50 min	$P_e = -0.00084t^2 + 0.0912t - 1.4424$	0.99
<i>Above 4 h</i>		
0–10 minutes	$P_e = 0.0024t^2 + 0.036t$	1.00
15–30 min	$P_e = 0.00012t^2 - 0.0072t + 0.7272$	0.85
35–55 min	$P_e = -0.00072t^2 + 0.0792t - 0.12744$	1.00

power output is 40–50 min and increases with the interval between shutdown and start-up.

Single polynomial equations could not provide good fits to the average electricity curves. We therefore had to separate the curves into two or three segments where applicable. **Table 2** shows the duration (t in minutes), correlation equations and coefficients for electricity output modelling.

3.3. Thermal power output

Fig. 6 shows plots of thermal power generated by the μ -CHP unit for different start-up intervals after shutdown. It is seen that all intervals have the same profile although the thermal power output for 1–2 h and 2–4 h intervals increases much more rapidly than those of less than 1 h and above 4 h within the first 10 min. After this time, the thermal power output increases at a slower rate, fluctuating about the mean before peaking at about 7.8 kW. The time taken to attain optimal thermal power output increases with the interval

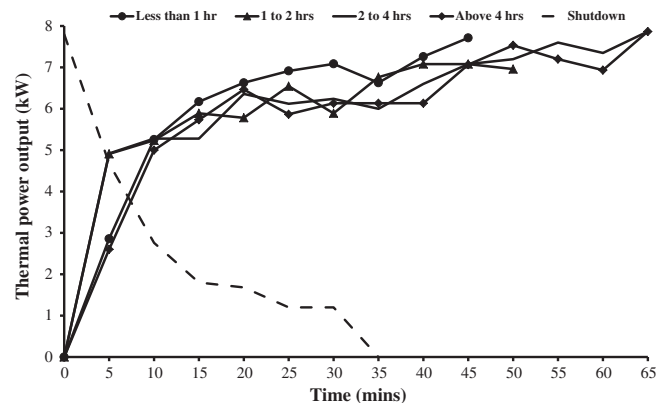


Fig. 6. Thermal power output for shutdown and different start-up times after shutdown.

Table 3

Duration, correlation equations and coefficients for thermal power output modelling.

Duration	Equation	R^2
Less than 1 h		
0–10 min	$P_{th} = -0.0096t^2 + 0.6168t$	1.00
15–30 min	$P_{th} = 0.00024t^3 - 0.024t^2 + 0.6984t + 0.102$	0.91
1–2 h		
0–5 min	$P_{th} = 0.098168t$	1.00
10–45 min	$P_{th} = -0.0504t + 4.8996$	0.83
2–4 h		
0–5 min	$P_{th} = 0.984t$	1.00
10–65 min	$P_{th} = 0.0444t + 4.896$	0.90
Above 4 h		
0–10 min	$P_{th} = 0.5196t$	1.00
15–65 min	$P_{th} = 0.0408t + 4.9692$	0.79
Shutdown		
5–20 min	$P_{th} = -0.0006t^3 + 0.0408t^2 - 0.9108t + 8.3328$	0.99
25 min	$P_{th} = -0.006t^2 + 0.2064t - 0.2544$	0.93
30–35 min	$P_{th} = -0.2328t + 8.1672$	1.00

between shutdown and start-up. During shutdown, thermal power dissipation reduces from 7.8 kW to 0 with a total of 13.1 kW of heat released from the μ -CHP unit over a period of 35 min.

Again, since single polynomial equations could not provide good fits to the average thermal power output curves, we separated the curves into two or three segments. Table 3 shows the duration (t in minutes), correlation equations and coefficients for thermal power output modelling.

3.4. Verification of the empirical model

Using the polynomial equations in Table 3, the modelled thermal and electrical power output during start-up and shutdown were verified against the measured 5-min performance data obtained from the field trial. This was deemed necessary to ensure that the modelled output for the different phases during start-up and shutdown were within acceptable error limits.

In order to quantify variations between predicted and measured values, two statistical parameters notably percentage mean absolute error (PMAE); and percentage mean error (PME) were used. PMAE evaluates the percentage mean of the sum of absolute deviations arising due to both over-estimation and under-estimation of individual observations. The PME evaluates the percentage mean of the sum of errors of individual observations. A negative value of PME indicates a net underestimation while a positive value indicates a net overestimation of the modelled values. PMAE and PME are given in Eqs. (5) and (6) as

$$PMAE = \frac{100}{N} \sum_{i=1}^N \frac{|m_i - M_i|}{M_i} \quad (5)$$

and

$$PME = \frac{100}{N} \sum_{i=1}^N \frac{m_i - M_i}{M_i} \quad (6)$$

N is the total number of observations while m_i and M_i are the i th modelled and measured values, respectively.

Table 4 shows the PMAE and PME for electrical and thermal power output for different start-up time intervals after shutdown. The modelled values for each observation in Eqs. (5) and (6) were obtained from the polynomials listed in Tables 2 and 3 while the measured values at 5-min intervals were obtained from field trial data. It is seen that the model has maximum PMAE of 5.34% and 4.38% for electrical and thermal power output respectively. The model overestimates electrical power output for start-up times between 2 and 4 h and underestimates for the other three intervals.

Table 4

PMAE and PME for electrical and thermal power outputs for different time intervals.

Start-up time	Electrical power output		Thermal power output	
	PMAE (%)	PME (%)	PMAE (%)	PME (%)
Less than 1 h	5.34	−0.29	2.73	2.27
1–2 h	1.16	−0.26	2.97	−0.39
2–4 h	1.62	1.56	3.40	−0.14
Above 4 h	2.97	−2.97	4.38	0.06
Shutdown			1.24	3.34

For thermal power output, the model overestimates for start-up times less than 1 h and above 4 h as well as during shutdown while it underestimates the thermal power output for start-up times between 1 and 2 h and between 2 and 4 h.

4. μ -CHP model implementation and validation

The structure of the model is such that if the heat demand profile (i.e. the times when heat is required and when the heat demand is zero) of a dwelling is known, the model predicts the thermal and electrical power output from the μ -CHP unit. In order to accurately predict the thermal and electrical power output when the μ -CHP unit is running, the start-up characteristics for the four start time intervals after shutdown notably less than 1 h; 1–2 h; 2–4 h; and above 4 h were programmed in separate loops.

Based on the duration between start-up and the last shutdown, the MatLab program decides on what set of equations (see Tables 2 and 3) to use to predict the thermal and electrical power output. The μ -CHP unit reaches continuous operation after the start-up period has elapsed. At the end of continuous operation, the shutdown characteristic of the unit is modelled and the thermal output determined. The electrical power output drops to zero when there is no heat demand. The program continues running until the entire heat demand profile has been satisfied. Fig. 7 shows a flow diagram of the implementation of the program for the μ -CHP unit.

The μ -CHP system model was run using the heat demand data for a domestic dwelling over a month. The model predicted both instantaneous and total values of thermal and electrical power generated by the μ -CHP unit. Instantaneous values of both measured and modelled electrical and thermal power output in 5-min intervals are plotted in Figs. 8 and 9. It is seen in Fig. 8 that the modelled and measured electrical power output showed good agreement during start-up and continuous operation with the model slightly over-predicting during shutdown. It is seen from Fig. 9 that the modelled and predicted values of thermal power output show good agreement during start-up and shutdown. However, during continuous operation the model predicts the mean thermal power output.

In order to quantify the accuracy of the model's predictions, the errors between modelled and measured total electrical and thermal power output by the μ -CHP unit were evaluated while the PMAE and PME were used to quantify the maximum and cumulative errors respectively. The data used to validate the model was collected from the Carbon Trust Accelerated Programme. The data was from the same μ -CHP unit installed in a different dwelling.

The results and errors between the modelled and measured values over a period of one month measured in 5-min intervals are shown in Table 5. The model over-predicted the total electrical energy output with cumulative, mean absolute and mean error of 1.4%, 7.4% and 4.7% respectively. The model over-predicted the total thermal energy output with cumulative, mean absolute and mean error of 4.2%, 7.2% and 3.7% respectively.

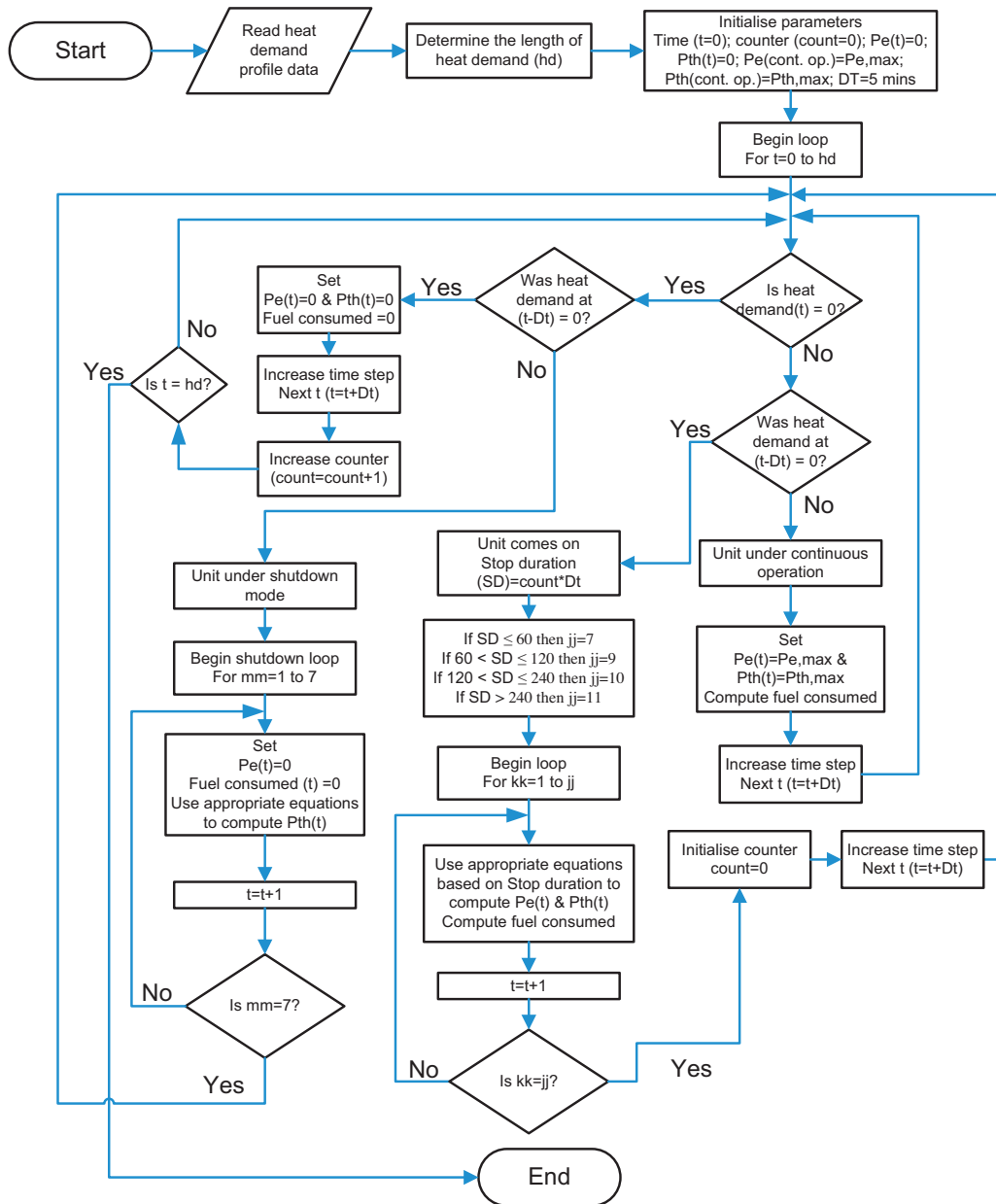


Fig. 7. Flow chart for the energy performance model of the μ -CHP unit.

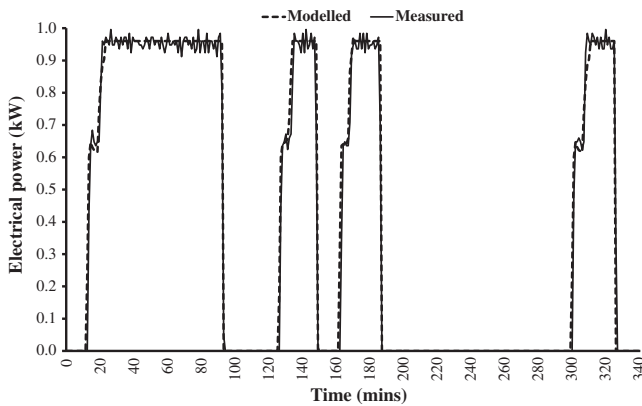


Fig. 8. Modelled and measured electrical power output.

5. Comparison of SE μ -CHP models

There are very few simulation models for combustion based SE μ -CHP units suitable for use in whole-building simulation programmes. Accurate and practical simulation models for SE μ -CHP units are necessary to assess their techno-economic feasibility in different buildings. One such model that has been widely used is the model developed by Kelly and Beausoleil-Morrison [16] within Annex 42 of the International Energy Agency's Energy Conservation

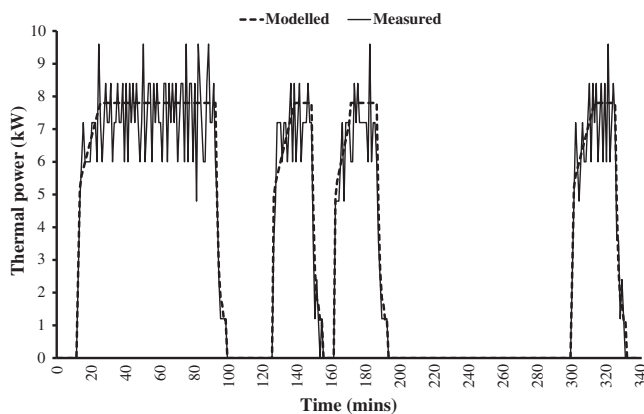
Table 5

Results and errors between modelled and measured values for 1 month's data.

	Total (kWh)		Cumulative error (%)	PMAE (%)	PME (%)
	Measured	Modelled			
Electrical energy	2090.6	2119.3	1.4	7.4	4.7
Thermal energy	17,521.5	18,256.1	4.2	7.2	3.7

Table 6Comparison between Models 1 and 2 for SE μ -CHP units.

Parameter	Model 1	Model 2
Time interval	5 min	10 s
Start-up	Cold and hot start are identified by four time intervals (<1 h, 1–2 h, 2–4 h and above 4 h) Electrical and thermal power output for each time interval are expressed as a function of the time after start-up	Cold and hot start are identified by the unit's temperature Fuel mass flow and electrical power output are expressed as a function of the engine and room temperatures
Steady-state (continuous operation)	Considers the thermal and electrical power as well as the thermal and electrical efficiencies to be constant	Represents the thermal and electrical efficiencies in a functional form that depends on the coolant flow rate, coolant inlet temperature and net electrical power output. The thermal power output is computed from the fuel mass flow and thermal efficiency
Shut-down	Provides an expression to compute the quantity of heat dissipated Ignores the electrical power generated due to the long time interval between measurements	Ignores the quantity of heat generated during this period since the quantity of fuel flow is zero Provides an expression to evaluate the electrical power generated during shut-down
General	Computes the electrical and thermal power output and uses the corresponding efficiencies to compute the quantity of fuel used.	Computes the electrical power output, fuel mass flow and thermal efficiency

**Fig. 9.** Modelled and measured thermal power output.

in Buildings and Community Systems Programme (IEA/ECBCS). Using an experimental data set collected at 10-s intervals, Lombardi et al. [17] proposed improvements to the model's treatment of steady-state, start-up and shut-down operations. In this section, the model presented in this paper for a WhisperGen SE μ -CHP Mk 4 unit (referred to as Model 1) is compared and contrasted against the improved Annex 42 model (referred to as Model 2).

5.1. Similarities

- Both Models 1 and 2 are empirical and dependent on data collected from the operation of a single SE μ -CHP unit.
- Both Models 1 and 2 treat steady-state and transient (start-up and shut-down) operating conditions.

5.2. Differences

The main differences between the two models are based on the number of variables used and the way in which the three operating modes start-up, steady-state and shut-down are treated. A summary of the differences between the two models is presented in Table 6.

6. Conclusions

A dynamic energy model that can be used to predict both thermal and electrical power output was developed for a WhisperGen Stirling engine μ -CHP Mk 4 unit (with a net electrical power output

of 1 kW and a net thermal output of 8 kW) and validated using field trial data. Using a heat demand profile for a house, the model over-predicted the total electrical and thermal power output from the μ -CHP unit by 1.4% and 4.2% respectively. The model predicted the electrical power output with a mean absolute error and mean error of 7.4% and 4.7% respectively. The thermal power was predicted with a mean absolute error and mean error of 7.2% and 3.7% respectively.

The model developed in this paper is based on empirical equations which are specific to the WhisperGen Stirling engine Mk 4 μ -CHP unit. The validate model can be used as a tool by installers to predict the thermal and electrical power output of the WhisperGen Stirling engine Mk 4 μ -CHP unit using the heat demand profile of a given dwelling. It can also be used by policy makers to explore the suitability of μ -CHP units in different house types for large-scale deployment of the Mk 4 μ -CHP. The methodology developed in this paper can easily be replicated for other μ -CHP technologies.

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